Bulk viscous effects near the QCD critical point

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A. Monnai, S. Mukherjee and YY, in preparation

Opportunities for Exploring Longitudinal Dynamics in Heavy Ion Collisions

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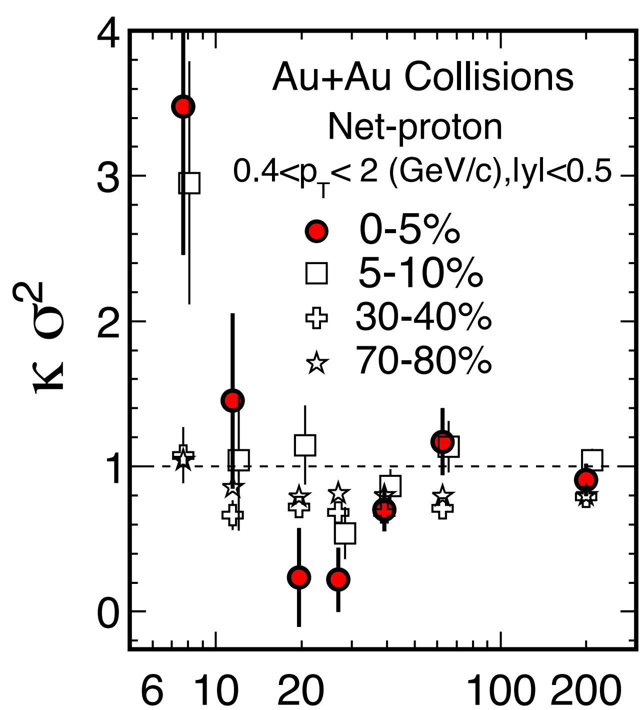
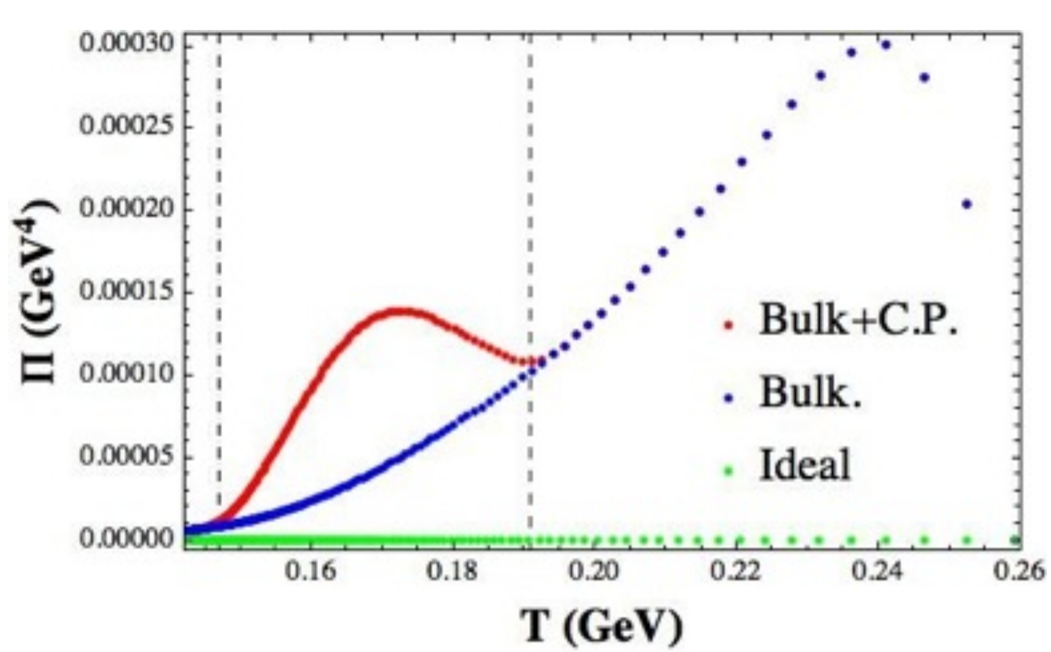
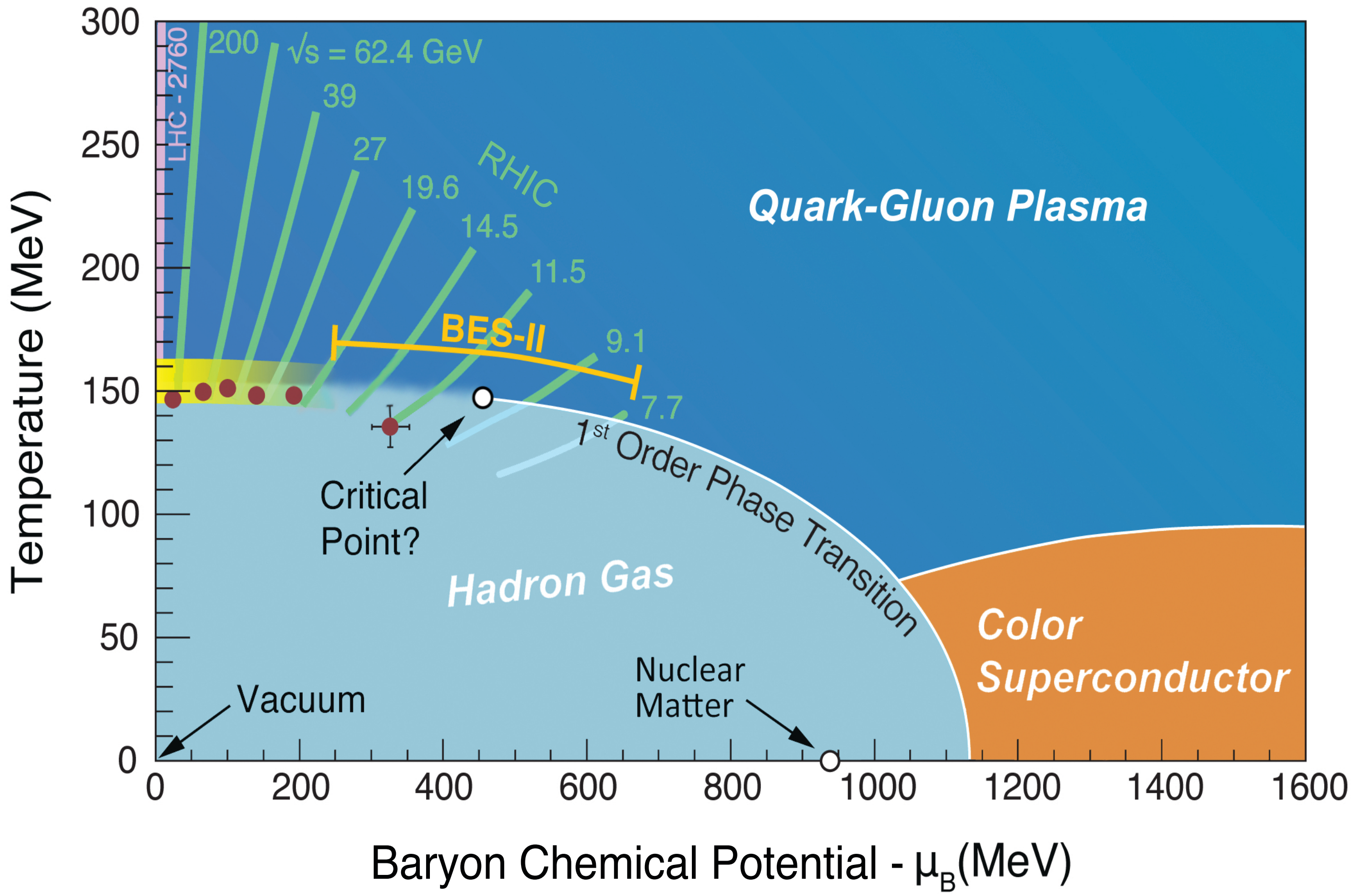
Highlights of the talk

- Bulk viscous pressure is very sensitive to the presence of the QCD critical point. $\Pi \sim \xi^3$
- Sizable critical fluctuations should be accompanied with influence due to enhanced bulk viscous pressure.
- Exploratory study with 1+1 hydro.





QCD phase diagram and the critical point



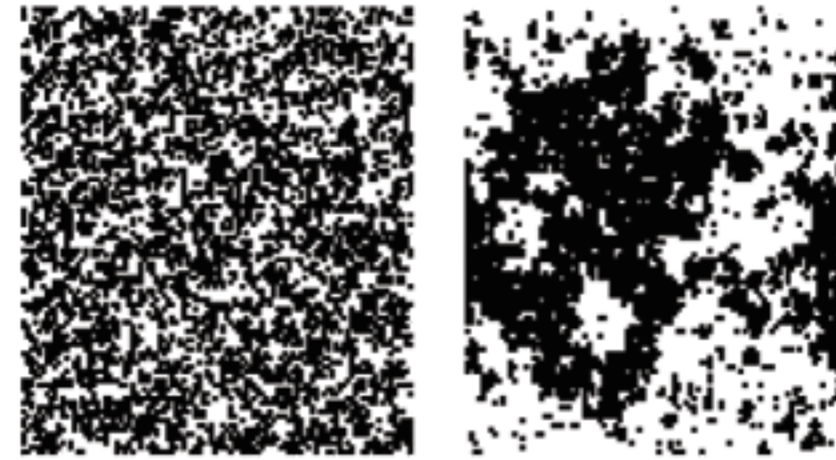
A sketch of the QCD phase diagram.

- QCD critical point: a distinctive feature in QCD phase diagram. (Its location is *unknown* theoretically).
- Beam Energy Scan Program: search for QCD critical point experimentally.

Correlation length and observables

- Correlation length $\xi_{\rm eq}$ of critical modes universally grows and becomes divergent near a critical point.

- Quantities with strong dependence on correlation length: important for search for QCD critical point.

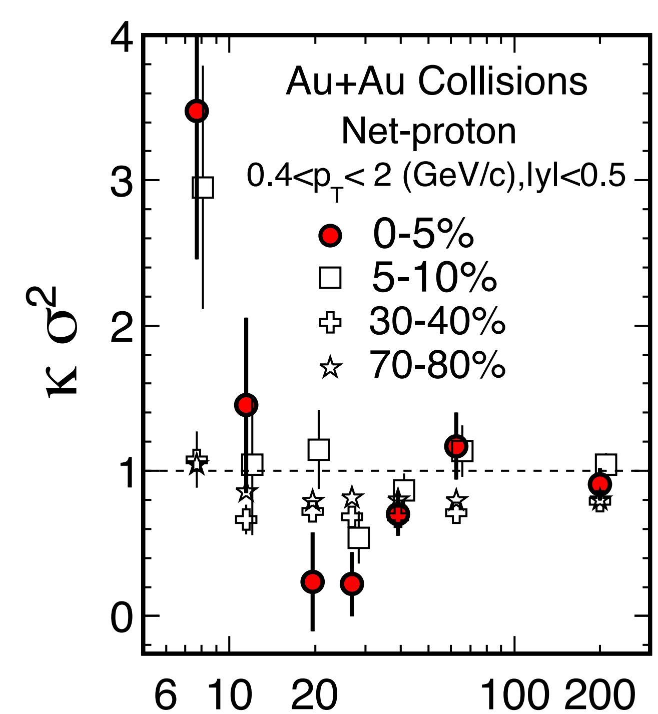


$T < T_c$ $T \approx T_c$

Spin configuration of Ising model

- Well-studied examples: fluctuation observables such as cumulants of net-proton numbers.

- Cumulants data is consistent with expectation due to critical fluctuation. Any other observables sensitive to the critical point?



Universal critical dynamics of the QCD critical point: I

- Both static properties (fluctuation) and dynamical properties (transport) universally scale with the correlation length.
- Two types of degrees of freedom:
  - short-range: rapid equilibration.
  - long-range (critical mode) : equilibrate diffusively and slowly (critical slowing down).
- Relaxation time of the critical mode: $\tau_\sigma \sim \xi_{\rm eq}^z$ with z>0 dynamic critical exponent.

Universal critical dynamics of the QCD critical point: II

- Dynamical universality class (Hohenberg-Halperin, Rev. Mod. Phys, 1977) : long distance dynamics should be matched to hydrodynamics.

- Universality class of QCD critical point: model H (Son-Stephanov, 2004), i.e. the same dynamic universality as Liquid-Gas transition.

$$z = 3 + \mathcal{O}(\epsilon)$$

- Transport coefficients scale with correlation length:

$$\text{(Baryon) conductivity}\, \sigma \sim \xi_{\text{eq}}\,,$$
$$\text{diffusive constant}\, D \sim \xi_{\text{eq}}^{-1}\,,$$
$$\text{shear viscosity}\, \eta \sim \xi_{\text{eq}}^{\mathcal{O}(\epsilon)}$$

- Bulk viscosity strongly depends on ξ_{eq} : $\zeta \sim \xi_{\text{eq}}^3$ (Onuki, 1997; Karsch-Kharzeev-Tuchi, PLB 2008; Moore-Saremi, JHEP 2008)

This talk: effects of bulk viscous pressure on search for QCD critical point.

- Introduction and motivations.
- Bulk viscous pressure near the QCD critical point
- Longitudinal expansion near the QCD critical point
- Summary and outlook

Bulk viscous pressure and bulk viscosity

- Bulk viscous pressure: non-equilibrium contribution to the effective pressure :

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + (p - \Pi)\,\Delta^{\mu\nu} + \text{shear viscous term}$$

$$p_{eff} \equiv p_{eq} - \Pi$$

- First order hydro. (Navier-Stokes limit):

$$\Pi \to \Pi_\zeta \equiv \zeta \partial_\mu u^\mu \qquad\qquad \Pi \sim \zeta \overset{?}{\sim} \xi_{eq}^3$$

- Growth of bulk pressure is limited by finite time effects: (c.f. Song-Heinz, 2009)

$$(u^\mu \partial_\mu)\,\Pi \sim \partial_\tau \Pi = \frac{1}{\tau_\Pi}\left[\,\Pi_\zeta - \Pi\,\right]$$

Israel-Stewart Theory

- Key input: the behavior of τ_Π near a critical point (unknown).

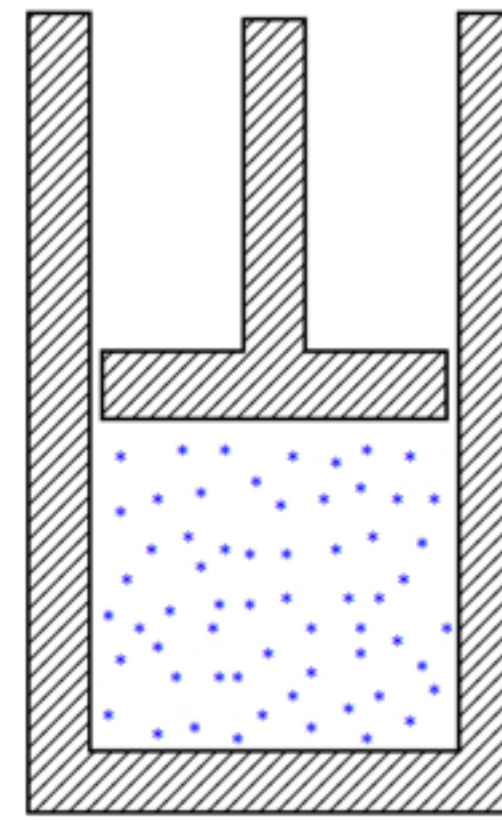
Bulk viscosity and τ_Π near the QCD critical point

$$\zeta \sim \text{equilibration of p under compression (decompression)}$$

- Pressure depends on the critical mode:

$$\text{equilibration time of pressure } \sim \tau_\sigma$$

$$\zeta \sim \tau_\sigma \sim \xi_{eq}^3$$



$$\tau_\Pi \sim \text{equilibration of p under a perturbation } \delta p$$

$$\text{IS Theory} \rightarrow \partial_\tau \delta p = -\frac{\delta p}{\tau_\Pi}$$

- New result (A. Monnai, S. Mukherjee and YY, in preparation):

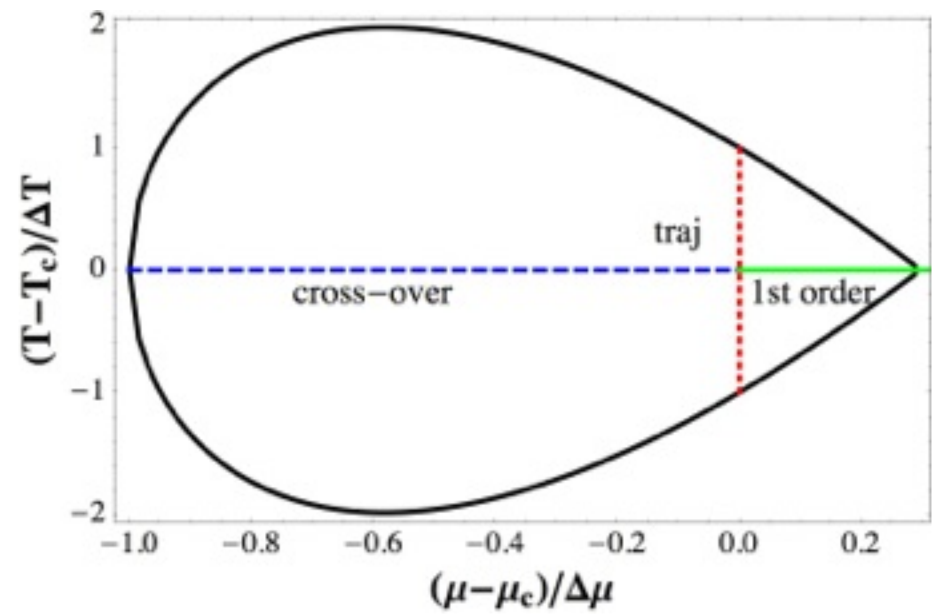
$$\tau_\Pi \sim \tau_\sigma \sim \xi_{\rm eq}^3$$

- Causality constrain (Romatschke. 2009):

$$\frac{dc_s(k)}{dk} \leq 1 \rightarrow \frac{\zeta}{\tau_\Pi} = \text{bounded}$$

A toy model

- Solving relaxation equation along a trajectory passing the critical regime:

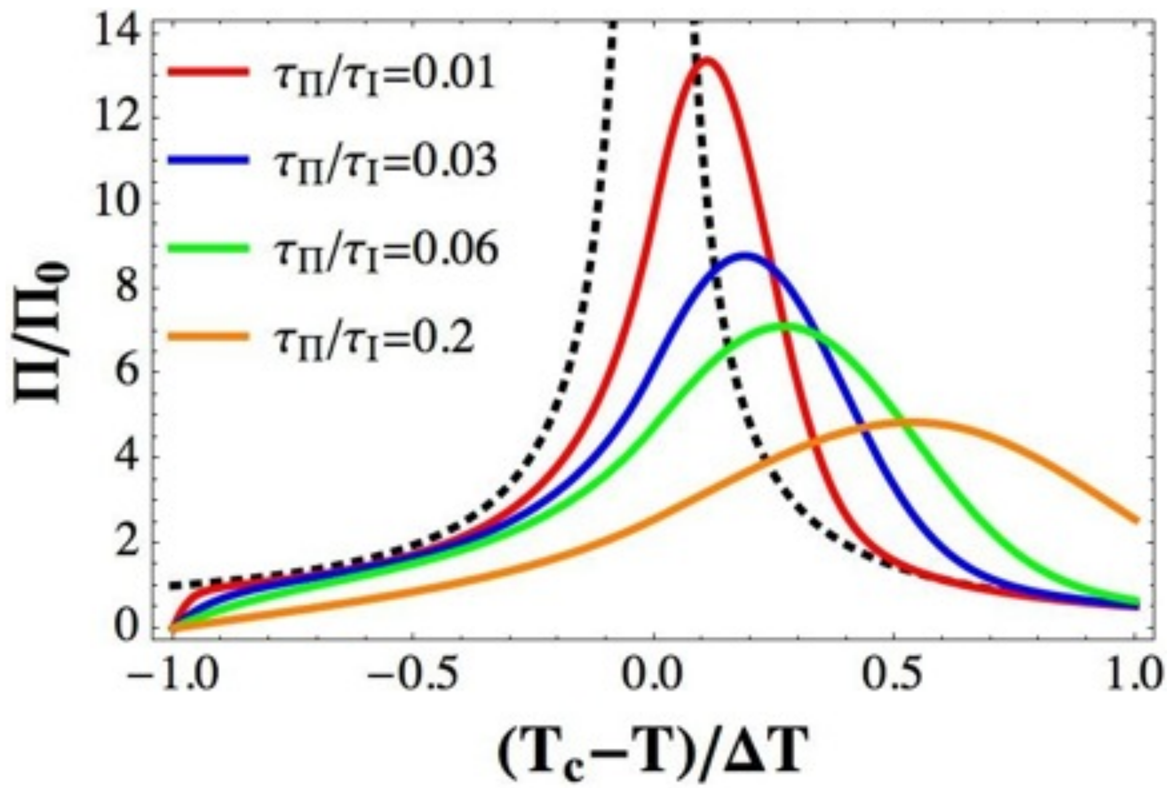


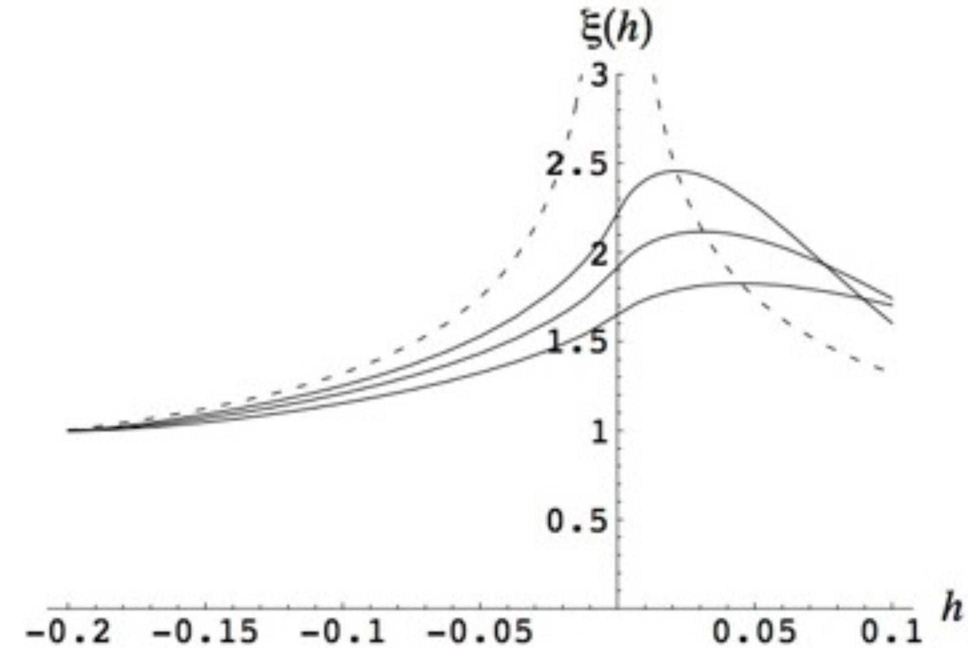
$$u^\mu \partial_\mu \Pi = \frac{1}{\tau_\Pi}\left[\zeta\left(\partial_\mu u^\mu\right) - \Pi\right] \rightarrow \partial_\tau \Pi = \frac{1}{\tau_\Pi}\left[\zeta\left(\frac{n_V}{\tau}\right) - \Pi\right]$$

- Implementing critical dynamics:

$$\zeta = \zeta_0 \left(\frac{\xi_{\rm eq}}{\xi_0}\right)^3 \qquad \tau_\Pi = \tau_{\Pi,0}\left(\frac{\xi_{\rm eq}}{\xi_0}\right)^3$$

Evolution of bulk viscous pressure





Evolution of non-equilibrium correlation length (Berdnikov-Rajagopal, 2000)

$$\partial_\tau \xi = (\xi_{eq} - \xi)/\tau_\sigma$$

- Finite time effects:
  - delay the growth of Π .
  - Preserve critical behavior.
- Similar to the evolution of non-equilibrium length.

Bulk viscous pressure and (non-equilibrium) correlation length

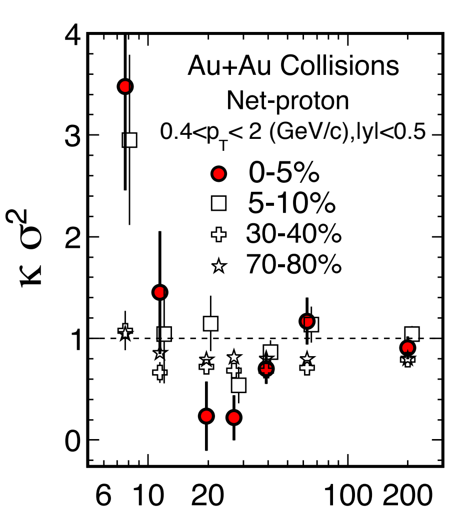
- The growth of ξ is accompanied with the growth of Π (tested numerically) .

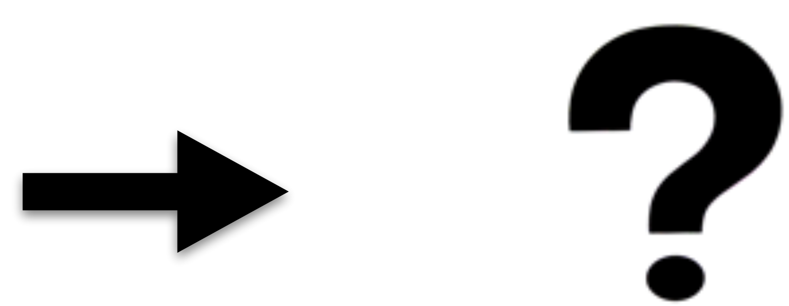
$$\Pi \sim \xi^3$$

- The critical behavior $\zeta \sim \tau_\Pi \sim \tau_\sigma \sim \xi_{\rm eq}^3$ are controlled by the same physics. (critical slowing down).

- Important implications:

Sizable critical fluctuations in data





Sizable effects due to Π

Observables sensitive to bulk viscous pressure

- Enhanced bulk pressure will reduce the effective pressure :

$$p_{eff} \equiv p_{eq} - \Pi$$

- If Π is too large thus $p_{eff} < 0$: cavitation (hydro. is broken)

- Within hydro. (IS theory): influence flow.

$$\partial_\mu T^{\mu\nu} = 0 \xrightarrow{\text{In fluid rest-frame}} \partial_t \vec{v} \propto \nabla p_{eff}$$

- Particle distribution at freeze-out:

$$\delta f \propto \frac{p_T^2}{T^2} \frac{\Pi}{\epsilon + p}$$

- Quantitative study: hydro. simulation with critical dynamics.

- Introduction and motivations.
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Set-up

- First attempt to study bulk viscous effects near the QCD critical point (A. Monnai, S. Mukherjee and YY, in preparation).

- 1+1 Israel-Stewart theory including baryon density.

  - E.O.S: lattice QCD with Taylor expansion.

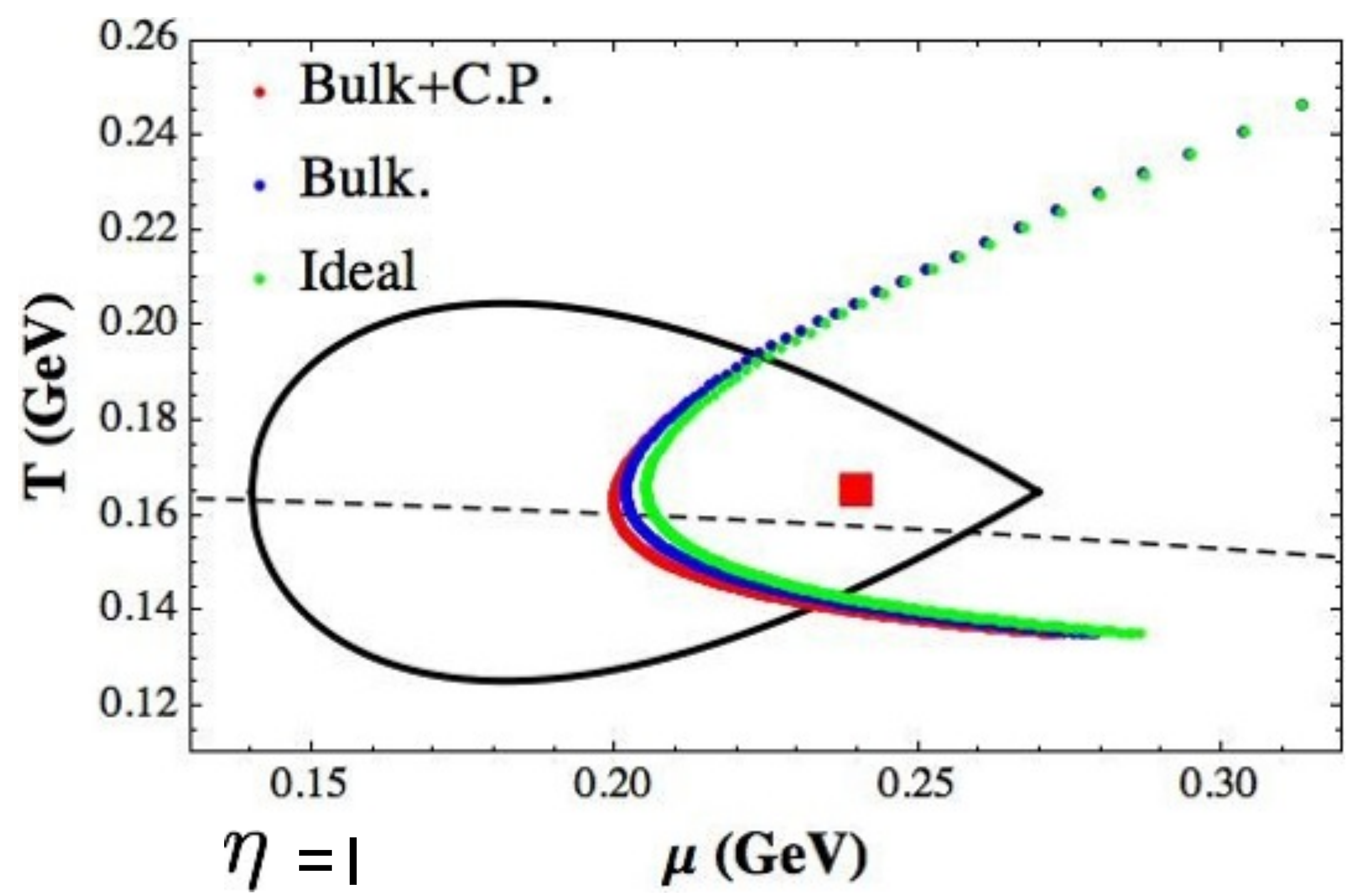
  - Initial condition: CGC (energy density) + Valence quark dist. (baryon density).

  - Linear mapping to Ising model

Preliminary Results I

- Run at 17 GeV, *assuming* (cross-over) side of the critical regime is probed. (Disclaimer: neither fitting the data or making prediction).

- Three different inputs:
    - Ideal hydro.
    - Bulk term without critical enhancement
    - Bulk term + critical enhancement



η =1

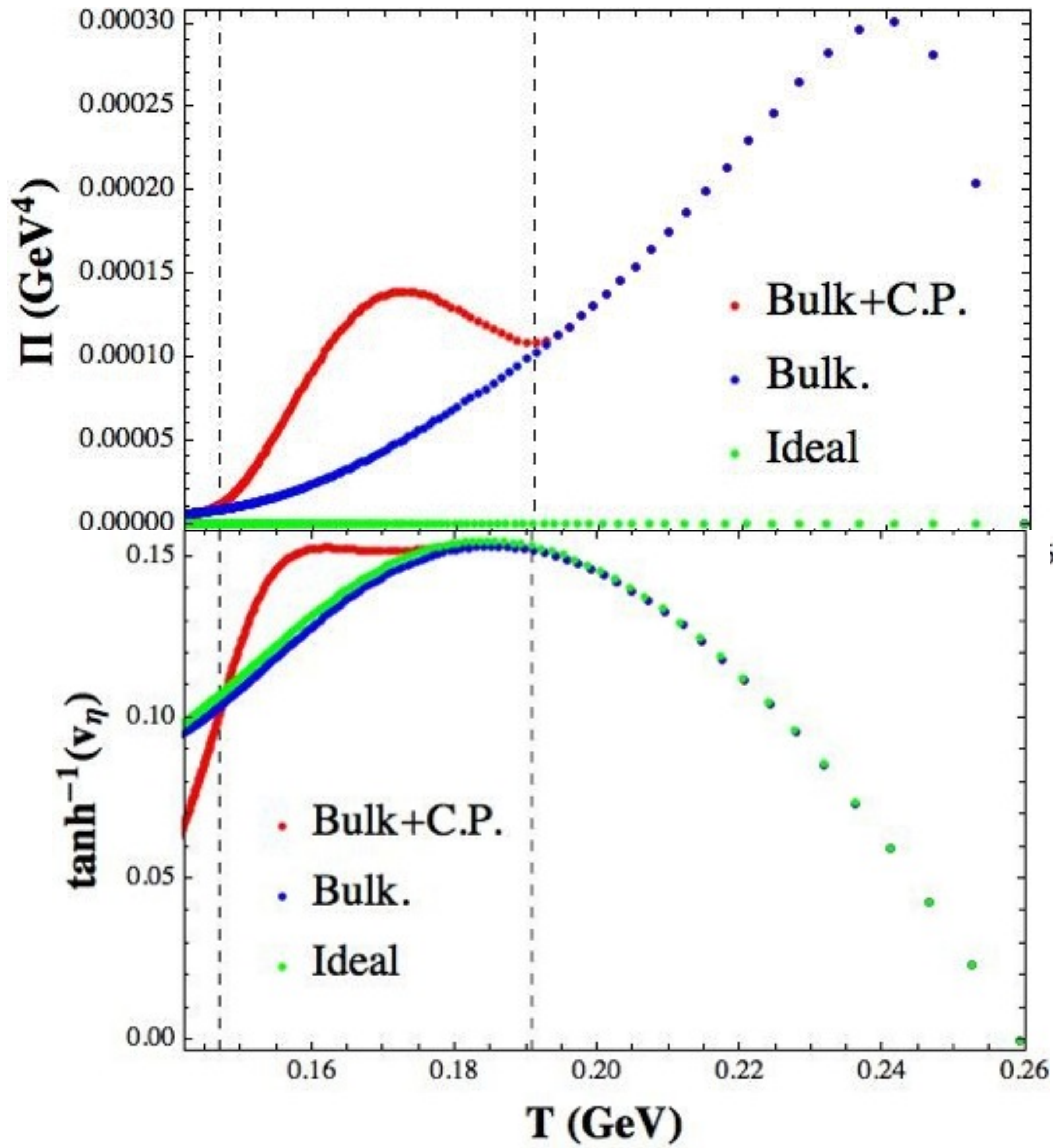
$$\zeta = \zeta_0 \left(\frac{\xi_{\rm eq}}{\xi_0} \right)^3 \qquad \tau_\Pi = \tau_{\Pi,0} \left(\frac{\xi_{\rm eq}}{\xi_0} \right)^3$$

Bulk viscous pressure and longitudinal flow η =1

- Enhanced bulk viscous pressure in the critical regime.
- Change of flow is frozen.

$$\partial_t \vec{v} \propto \nabla p_{eff}$$

- Particle distribution, in progress.



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Summary

- Explore the critical dynamics and its implication on search for QCD critical point.

- Bulk viscous pressure is correlated and sensitive to the growth of correlation length.

- Flow observables: sensitive to the reduction of effective pressure. (Complementary to fluctuation observables, higher statistics)

- Exploratory study with 1+1 hydro underway.

Outlook

- Directed flow observables: reaches minimum for smaller pressure (the softest point due to bulk viscous pressure?).

- Improve the domain of validity of hydro near a QCD critical point (in progress with M. Stephanov).

- Critical behavior of second order hydro. coefficients (extension of the classic work by Hohenberg-Halperin).

- Many interesting questions ahead!

$$\partial_t \vec{v} \propto \nabla p_{eff}$$